2023 泛函分析及空间理论天元暑期研讨班

Concentration day:
young researchers on noncommutative analysis I

July 17, 2023 Zheng Xin Building, Room 226

9:00-11:30

A local smoothing estimate on quantum Euclidean space

Liang WANG（王亮） Wuhan University

In this talk, we will review some developments on the local smoothing conjecture in Euclidean space. Then we will introduce a local smoothing estimate on quantum Euclidean space based on a operator-valued local smoothing estimate. This is a joint work with G.Hong and X.Lai.

Recent progress on Schatten classes and Riesz transform commutators

Zhijie FAN（范智杰） Guangzhou University

This talk is about recent progress on Schatten classes and non-Euclidean Riesz transform commutators. By developing a new approach to bypass the use of Fourier analysis and the standard dyadic structure of Euclidean space, we show that the Schatten norm of several kinds of non-Euclidean Riesz transform commutators can be characterized in terms of Besov norms of the symbol.

Noncommutative von Bahr-Esseen inequality and a noncommutative nonindependent law of large numbers

Enli CHEN（陈恩立） Wuhan University

The von Bahr-Esseen inequality is a powerful martingale inequality established in 1965. This inequality has a great deal of applications in many fields. Using the noncommutative Burkholder-Gundy inequality, we establish the noncommutative von Bahr-Esseen inequality. And then, we use this noncommutative inequality to extend the Marcinkiewicz-Zygmund type strong law of large numbers for nonindependent random variables to the noncommutative setting. This is a joint work with Guixiang Hong and Yanqi Qiu.

14:30-17:00

Weighted endpoint estimates of operator-valued singular integrals

Wenfei FAN（范雯霏） Central South University

The Calderón-Zygmund theory of singular integral operators plays a central role in harmonic analysis. In this talk, we are concerned about the weighted inequalities of noncommutative Calderón-Zygmund operators, including the weighted weak (1,1) estimates for maximal operators and square functions, and weighted $H_1 - L_1$ inequalities.

Cesàro summability of noncommutative Vilenkin-Fourier series

Tiantian ZHAO（赵甜甜） Central South University

Let $\mathcal{R}$ be the hyperfinite II_1 factor, and let $(\sigma_n^{\mathcal{R}})_{n\geq 1}$ be Cesàro means of the noncommutative Vilenkin-Fourier series. We first establish the following noncommutative weak type maximal inequality

$$\|(\sigma_n^{\mathcal{R}}(x))_{n\geq 1}\|_{\Lambda_{1,\infty}(\mathcal{R},\ell_\infty)} \leq c\|x\|_{L_1(\mathcal{R})}.$$

As a consequence, $\sigma_n^{\mathcal{R}}(x)$ converges bilaterally almost uniformly to $x \in L_1(\mathcal{R})$. Moreover, considering the subsequence $(\sigma_{n_k}^{\mathcal{R}})_{k\geq 1}$ of $(\sigma_n^{\mathcal{R}})_{n\geq 1}$, where $(n_k)_{k\geq 1}$ is a lacunary sequence, we establish the related noncommutative asymmetric maximal inequalities:

$$\|(\sigma_{n_k}^{\mathcal{R}}(x))_{k\geq 1}\|_{\Lambda_{p,\infty}(\mathcal{R},\ell_\infty^c)} \leq c_p\|x\|_{H_p^c(\mathcal{R})},\ 1 \leq p \leq 2$$

and

$$\inf_{x=x^c+x^r}\left\{\|(\sigma_{n_k}^{\mathcal{R}}(x^c))_{k\geq 1}\|_{L_p(\mathcal{R},\ell_\infty^c)} + \|(\sigma_{n_k}^{\mathcal{R}}(x^r))_{k\geq 1}\|_{L_p(\mathcal{R},\ell_\infty^r)}\right\} \leq c_p\|x\|_{L_p(\mathcal{R})}, \quad 1 < p < 2.$$

Consequently, $\sigma_{n_k}^{\mathcal{R}}(x)$ converges column (resp. row) almost uniformly to x whenever $x \in H_1^c(\mathcal{R})$ (resp. $H_1^r(\mathcal{R})$); $\sigma_{n_k}^{\mathcal{R}}(x)$ converges column + row almost uniformly to x whenever $x \in L_p(\mathcal{R})$ with $1 < p < 2$. The primary strategy in our proof is to explore an appropriate noncommutative square function estimate via the atomic decomposition of noncommutative martingale Hardy space, and the very recent advance of the noncommutative Calderón-Zygmund decomposition established by Cadilhac, Conde-Alonso and Parcet.

Noncommutative weak-L^∞ and BMO

Yahui Zuo（左雅慧） Central South University

Bennett, DeVore and Sharpley introduced the weak-L^∞ space and studied the relationship between weak-L^∞ and BMO spaces (Ann of Math.113: 601-611, 1981). We extend their result to the noncommutative setting. Furthermore, we obtain weak-L^∞ estimates of the Stein inequalities and the Dual Doob inequalities for noncommutative martingales.