

Hardy-Littlewood-Sobolev Inequality on Product Spaces

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Abstract

Product theory in harmonic analysis dates back to the time of Jessen, Marcinkiewicz and Zygmund as the strong maximal function was first investigated. Study of certain operators, commuting with a multi-parameter family of dilations, has seen little progress since the 1990s after a number of pioneering works accomplished. In particular, the area remains largely open for fractional integrals.

We study the regularity of fractional integral

$$\int_{\mathbb{R}^n} f(y) \left(\frac{1}{\mathbf{V}(x, y)} \right)^{1-\frac{\alpha}{n}} d\mu(y) \quad (0 < \alpha < n)$$

where $\mathbf{V}(x, y)$ is essentially the volume of the smallest rectangle centered on $x \in \mathbb{R}^n$ that contains $y \in \mathbb{R}^n$, with respect to a *rectangle* A_∞ measure μ .

As a result, we give an extension of the classical Hardy-Littlewood-Sobolev theorem to product spaces