PRINCIPAL EIGENVALUE OF AN ELLIPTIC OPERATOR WITH LARGE DEGENERATE ADVECTION

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Abstract: In this article, we study, as the coefficient $s \to \infty$, the asymptotic behavior of the principal eigenvalue of the eigenvalue problem

$$-\varphi''(x) - 2sm'(x)\varphi'(x) + c(x)\varphi(x) = \lambda(s)\varphi(x), \quad 0 < x < 1,$$

complemented by a general boundary condition. This problem is relevant to nonlinear propagation phenomena in reaction-diffusion equations. The main point is that the advection (or drift) term m allows natural degeneracy. For instance, m can be constant on $[a, b] \subset [0, 1]$. Depending on the behavior of m near the neighbourhood of the endpoints a and b, the limiting value could be the principal eigenvalue of

$$-\varphi''(x) + c(x)\varphi(x) = \lambda\varphi(x), \quad a < x < b,$$

coupled with Dirichlet or Neumann boundary condition at a and b. A complete understanding of the limiting behavior of the principal eigenvalue and its eigenfunction is obtained, and new fundamental effects of large degenerate advection and boundary conditions on the principal eigenvalue and the principal eigenfunction are revealed. In one space dimension, the results in the existing literature are substantially improved.