

**WEAK TYPE COMMUTATOR AND LIPSCHITZ ESTIMATES:
RESOLUTION OF THE NAZAROV-PELLER CONJECTURE**

ABSTRACT. Let \mathcal{M} be a semi-finite von Neumann algebra and let $f : \mathbb{R} \rightarrow \mathbb{C}$ be a Lipschitz function. If $A, B \in \mathcal{M}$ are self-adjoint operators such that $[A, B] \in L_1(\mathcal{M})$, then

$$\|[f(A), B]\|_{1, \infty} \leq c_{abs} \|f'\|_{\infty} \|[A, B]\|_1,$$

where c_{abs} is an absolute constant independent of f , \mathcal{M} and A, B and $\|\cdot\|_{1, \infty}$ denotes the weak L_1 -norm. If $X, Y \in \mathcal{M}$ are self-adjoint operators such that $X - Y \in L_1(\mathcal{M})$, then

$$\|f(X) - f(Y)\|_{1, \infty} \leq c_{abs} \|f'\|_{\infty} \|X - Y\|_1.$$

This result resolves a conjecture raised by F. Nazarov and V. Peller implying a couple of existing results in perturbation theory. Joint work with M. Caspers, D. Potapov and D. Zanin