General Information on the conference "Lie algebras and Related Topics"

Scientific Committee:

Shavkat Ayupov	V.I. Romanovsky Institute of Mathematics, Uzbekistan Academy of Sciences, Uzbekistan
Askar Dzhumadil'daev	National Academy of Sciences of the Republic of Kazakhstan, Kazakhstan
Zhaobing Fan	Harbin Engineering University, China
Vyacheslav Futorny	Southern University of Science and Technology, China
Yunhe Sheng	Jilin University, China

Local Organizing Committee:

Jinghao Huang	Institute for Advanced study of Mathematics, Harbin Institute of Technology
Karimbergen Kudaybergenov	Institute for Advanced study of Mathematics, Harbin Institute of Technology
Bakhrom Omirov	Institute for Advanced study of Mathematics, Harbin Institute of Technology

List of participants

1. Shavkat Ayupov, V.I. Romanovsky Institute of Mathematics, Uzbekistan Academy of Sciences, Uzbekistan
2. Chengming Bai, Chern Institute of Mathematics, China
3. Sofiane Bouarroudj, New York University Abu Dhabi, United Arab Emirates
4. Antonio Jesus Calderon Martin, University of Cadiz, Spain
5. Askar Dzhumadil'daev, National Academy of Sciences of the Republic of Kazakhstan, Kazakhstan
6. Zhaobing Fan, Harbin Engineering University, China
7. Vyacheslav Futorny, Southern University of Science and Technology, China
8. Xabier Garcia, University of Vigo, Spain
9. Iryna Kashuba, Southern University of Science and Technology, China
10. Pavel Kolesnikov, Novosibirsk State University, Russia
11. Jiefeng Liu, Northeast Normal University, China
12. Yaohua Liu, Southern University of Science and Technology, China
13. Haitao Ma, Harbin Engineering University, China
14. Farrukh Mashurov, Southern University of Science and Technology, China
15. Rosa Navarro, University of Extremadura, Spain
16. Zafar Normatov, Jiling University, China
17. Amir Fernández Ouaridi, University of Cadiz, Spain
18. Jhon Alexander Ramirez, Southern University of Science and Technology, China
19. Utkir Rozikov, V.I. Romanovsky Institute of Mathematics, Uzbekistan Academy of Sciences, Uzbekistan
20. Joao Schwarz, Southern University of Science and Technology, China
21. Yunhe Sheng, Jilin University, China
22. Fedor Sukochev, University of New South Wales, Australia
23. Tong Wu, Southern University of Science and Technology, China
24. Jinghao Huang, IASM of HIT, China
25. Karimbergen Kudaybergenov, IASM of HIT, China
26. Bakhrom Omirov, IASM of HIT, China

Program Schedule

16^{th} September, 2024

Chair: Antonio Jesus Calderon Martin

Time	Name of speaker & title of talk
9^{00}-10^{00}	Shavkat Ayupov *2-local derivations on von Neumann algebras and C*- algebras.*
10^{00}-11^{00}	Chengming Bai *Deformation families of Novikov bialgebras via differential antisymmetric infinitesimal bialgebras.*
11^{20}-12^{20}	Askar Dzhumadil'daev *Identities of n-Lie dialgebras and Transposed Poisson structures.*
12^{20}-14^{00}	***Break***
14^{00}-15^{00}	Haitao Ma *Geometric approach to Mirabolic Schur-Weyl Duality of type A.*
15^{20}-16^{20}	Sofiane Bouarroudj *Central extensions of restricted Lie superalgebras and classification of p-nilpotent Lie superalgebras in dimension 4.*
16^{20}-17^{20}	Iryna Kashuba *Free algebras in the Tits-Kantor-Koecher category.*

17^{th} September, 2024

Chair: Rosa Navarro

Time	Name of speaker & title of talk
9^{00}-10^{00}	Fedor Sukochev *Spectral asymptotics and scattering theory in the nilpotent Lie group setting.*
10^{00}-11^{00}	Utkir Rozikov *Flows of finite-dimensional algebras.*
11^{20}-12^{20}	Zhaobing Fan *Global bases, canonical bases and perfect bases for quantum Borcherds-Bozec algebras.*
12^{20}-14^{00}	***Break***
14^{00}-15^{00}	Yunhe Sheng *Homotopy theory of post-Lie algebras.*
15^{20}-16^{20}	Antonio Jesus Calderon Martin *A linear characterization of the (semi)simplity of arbitrary algebras.*
16^{20}-17^{20}	Xabier Garcia *A characterization of Lie algebras using ideals and subalgebras.*

18^{th} September, 2024

Chair: Sofiane Bouarroudj

Time	Name of speaker & title of talk
9^{00}-10^{00}	Pavel Kolesnikov *Differential envelopes of Novikov conformal algebras.*
10^{00}-11^{00}	Rosa Navarro *Solvable Lie and Leibniz superalgebras with a given nilradical.*
11^{20}-12^{20}	Jiefeng Liu *Differential calculus on Lie conformal algebroids.*
12^{20}-14^{00}	***Break***
14^{00}-17^{30}	***Free discussion***

19th September, 2024

Chair: Xabier Garcia

Time	Name of speaker & title of talk
9^{00}-10^{00}	Vyacheslav Futorny *Representations of map Lie superalgebras.*
10^{00}-11^{00}	Amir Fernández Ouaridi *Abelian subalgebras and ideals of maximal dimension in algebras.*
11^{20}-12^{20}	Farrukh Mashurov *On the contact and transposed Poisson algebras.*
12^{20}-14^{00}	***Break***
14^{00}-17^{30}	***Free discussion***

Title and Abstract for Talks

16th September, 2024

Shavkat Ayupov
(V.I. Romanovsky Institute of Mathematics, Uzbekistan Academy of Sciences, Uzbekistan)

2-local derivations on von Neumann algebras and C*- algebras.

Abstract: Given an algebra A, a linear mapping $T : A \to A$ is called *a homomorphism* (respectively, *a derivation*) if $T(ab)=T(a)T(b)$ (respectively, $T(ab)=T(a)b+aT(b)$) for all a, b in A. A one-to-one homomorphism is called an *automorphism.* A mapping Δ: A →A (not linear in general) is called *a 2-local automorphism* (respectively, *a 2-local derivation*) on A, if for every x,y in A there exists an automorphism $a_{x,y}$ (respectively, a derivation $d_{x,y}$ on A depending on x and y, such that

$\Delta(x)= a_{x,y}(x)$, $\Delta(y)= a_{x,y}(y)$ (respectively, $\Delta(x)= d_{x,y}(x)$ and $\Delta(y)= d_{x,y}(y)$).

The main problem concerning the above notions are to find conditions under which every 2-local automorphism or derivation automatically becomes an automorphism (respectively, a derivation). For general C*-algebras this problem is still open.

In the present talk we give a solution of this problem in the framework of von Neumann algebras and their abstract generalization – AW*-algebras (i.e. Kaplansky algebras). Also, we give a survey of corresponding results of other authors for certain classes of C*-algebras.

Chengming Bai
(Chern Institute of Mathematics, China)

Deformation families of Novikov bialgebras via differential antisymmetric infinitesimal bialgebras.

Abstract: We generalize S. Gelfand's classical construction of a Novikov algebra from a commutative differential algebra to get a deformation family (A, $\circ_q$) of Novikov algebras by an admissible commutative differential algebra, which ensures a bialgebra theory of commutative differential algebras, enriching the antisymmetric infinitesimal bialgebra. Moreover, a deformation family of Novikov bialgebras is obtained, under certain further condition. In particular, we obtain a bialgebra variation of S. Gelfand's construction with an interesting twist: every commutative and cocommutative differential antisymmetric infinitesimal bialgebra gives rise to a Novikov bialgebra whose underlying Novikov algebra is (A, $\circ_{-\frac{1}{2}}$) instead of (A, $\circ_0$) which recovers the construction of S. Gelfand. This is the joint work with Yanyong Hong and Li Guo.

Askar Dzhumadil'daev

(5.National Academy of Sciences of the Republic of Kazakhstan, Kazakhstan)

Identities of n-Lie dialgebras and Transposed Poisson structures.

Abstract: Polynomial identities for different kind of n-Lie dialgebras are studied. Corresponding Transposed Poisson structures are constructed.

Haitao Ma

(Harbin Engineering University, China)

Geometric approach to Mirabolic Schur-Weyl Duality of type A.

Abstract: We commence by constructing the mirabolic quantum Schur algebra, utilizing the convolution algebra defined on the variety of triples of two n-step partial flags and a vector. Subsequently, we employ a stabilization procedure to derive the mirabolic quantum gl_n. Then we present the geometric approach of the mirabolic Schur-Weyl duality of type A.

Sofiane Bouarroudj

(New York University Abu Dhabi, United Arab Emirates)

Central extensions of restricted Lie superalgebras and classification of p-nilpotent Lie superalgebras in dimension 4.

Abstract: We build the first and second restricted cohomology groups for restricted Lie superalgebras in characteristic p ≥ 3, modifying a construction by Yuan, Chen and Cao. We will explain how these groups capture some algebraic structures, such as extensions and derivations. Further, we apply this construction to classify p-nilpotent restricted Lie superalgebras up to dimension 4 over an algebraically closed field of characteristic p ≥ 3. This is a joint work with Quentin Ehret.

Iryna Kashuba

(Southern University of Science and Technology, China)

Free algebras in the Tits-Kantor-Koecher category.

Abstract: We study free algebras in the category of sl(2)-modules that are sums of copies of trivial and adjoint representations. This category is a home for Lie algebras which appear applying the celebrated Tits-Kantor-Koecher construction to Jordan algebras and therefore we call it the Tits-Kantor-Koecher category. The study of homological properties of free algebras (free associative, free commutative associative and free Lie algebra) is motivated by the conjecture of I. Kashuba and O. Mathieu that certain homologies of the free Lie algebra in TKK category vanish, which, if true, gives formulas for dimensions of homogeneous components of the free Jordan algebra. This is joint work with Vladimir Dotsenko.

17th September, 2024

Fedor Sukochev

(University of New South Wales, Australia)

Spectral asymptotics and scattering theory in the nilpotent Lie group setting.

Abstract: Scattering theory is one of the cornerstones of mathematical physics, and the scattering theory of elliptic differential operators on Euclidean space and on manifolds is well-understood. Recently there have been significant advances made in the study of hypoelliptic differential operators. This new understanding opens up the possibility to understand scattering theory for hypoelliptic operators. The simplest possible case, of left-invariant sub-Laplacians on stratified Lie groups, is an important test case for these ideas. I overview recent work with E. McDonald and D. Zanin where important foundational results in scattering theory were proved in the stratified Lie group setting.

Utkir Rozikov

(V.I. Romanovsky Institute of Mathematics, Uzbekistan Academy of Sciences, Uzbekistan)

Flows of finite-dimensional algebras.

Abstract: A flow of algebras is a specific type of continuous-time dynamical system where the states are finite-dimensional algebras with (cubic) matrices of structural constants that satisfy an analogue of the Kolmogorov-Chapman equation (KCE). In this talk, we present sufficient conditions for the multiplications of cubic matrices under which the corresponding KCE has a solution. Our conditions are primarily provided for the algebra of cubic matrices, considered with respect to a fixed multiplication of cubic matrices. Specifically, by adapting the theory of continuous-time Markov processes, we construct a class of flows of algebras given by the matrix exponent of cubic matrices.

Zhaobing Fan

(Harbin Engineering University, China)

Global bases, canonical bases and perfect bases for quantum Borcherds-Bozec algebras.

Abstract: By using new Kashiwara operators, we construct global bases for quantum Borcherds-Bozec algebra. Moreover, we introduce the primitive canonical bases, and prove it coincides with global bases. This is a joint work with Han, Kang and Kim.

Yunhe Sheng

(Jilin University, China)

Homotopy theory of post-Lie algebras.

Abstract: Guided by Koszul duality theory, we consider the graded Lie algebra of coderivations of the cofree conilpotent graded cocommutative cotrialgebra generated by a graded vector space V. We show that in the case of V being a shift of an ungraded vector space W, Maurer-Cartan elements of this graded Lie algebra are exactly post-Lie algebra structures on W. The cohomology of a post-Lie algebra is then defined using Maurer-Cartan twisting. The second cohomology group of a post-Lie algebra has a familiar interpretation as equivalence classes of infinitesimal deformations. Next we define a post-Lie-infty algebra structure on a graded vector space to be a Maurer-Cartan element of the aforementioned graded Lie algebra. Post-Lie-infty algebras admit a useful characterization in terms of L-infty-actions (or open-closed homotopy Lie algebras). Finally, we introduce the notion of homotopy Rota-Baxter operators on open-closed homotopy Lie algebras and show that certain homotopy Rota-Baxter operators induce post-Lie-infty algebras. This is a joint work with Andrey Lazarev and Rong Tang.

Antonio Jesus Calderon Martin
(University of Cadiz, Spain)

A linear characterization of the (semi)simplity of arbitrary algebras.

Abstract: We show that an arbitrary algebra A is semisimple if and only if it has zero annihilator and admits a weak-division linear basis. As a corollary, the simplicity of A is also characterized. With more detail, an algebra A is just a linear space (or arbitrary dimension) over an arbitrary base field K endowed with a bilinear map (called the product of A). The algebra A is said to be simple if its only ideals are 0 and A; and semisimple if it is a direct sum of simple ideals. The annihilator of A is the set of elements x in A such that xA+Ax=0, where juxtaposition denotes the product of A. We will show that A is semisimple if and only if t has zero annihilator and admits a linear basis B in such a way that any element of B satisfies a “division" property”". A linear characterization of the simplicity of A is also provided.

Xabier García

(University of Vigo, Spain)

A characterization of Lie algebras using ideals and subalgebras.

Abstract: Inspired by the categorical characterization of Lie algebras amongst all varieties of non-associative algebras using *algebraic exponents* [2], the following result was proved in [1]:

Theorem. Suppose that M is a non-trivial variety of non-associative algebras over a field of zero characteristic satisfying the following two conditions:

- every subalgebra of every free algebra is free;
- for every ideal I in every algebra, I^2 is also an ideal.

Then M is the variety of Lie algebras.

A variety satisfying the first condition is called *Nielsen-Schreier*, whilst a variety satisfying the second condition is said to be a *2-variety*.

In this talk we will first focus on giving a wide and intuitionistic idea about this result, together with its motivation and origins. Then we will explain the methods used to prove it, which include homological and computational

algebra, together with Gröbner bases for operads. This work is joint with Vladimir Dotsenko (Université de Strasbourg)

References.

[1] V. Dotsenko and X. García-Martínez. A characterisation of Lie algebras using ideals and subalgebras, to appear in *Bull. Lond. Math. Soc.* (2024).

[2] X. García-Martínez and T. Van der Linden. A characterisation of Lie algebras via algebraic exponentiation. *Adv. Math*., 341, (2019), 92 –117.

18th September, 2024

Pavel Kolesnikov

(Novosibirsk State University, Russia)

Differential envelopes of Novikov conformal algebras.

Abstract: Novikov algebras form a class of nonassociative algebras with one binary operation which emerged in various studies of I. Gel'fand, I. Dorfman [1] and A. Balinski, S. Novikov [2]. In both cases, the defining identities of Novikov algebras represent certain conditions on the coefficients of a rank 3 tensor that appears in functional analysis or in the study of differential equations. Namely, if one considers these coefficients as structure constants of a finite-dimensional algebra with a bilinear operation ○ then this algebra should satisfy the following identities:

(1) $x \circ (y \circ z) = y \circ (x \circ z)$,

(2) $(x \circ y) \circ z - x \circ (y \circ z) = (x \circ z) \circ y - x \circ (z \circ y)$.

Denote by Nov the class of all algebras satisfying (1), (2).

The structure theory of Novikov algebras has been studied in a series of papers by E. Zel'manov, M. Osborn, and X. Xu. On the other side of the spectrum, free Novikov algebras were constructed in [3]. In particular, the latest results imply an important observation that can be placed in a more general context as follows.

Let Var be a variety of algebras with bilinear operations μ_i. Recall that a *derivation* of an algebra $A \in$ Var is a linear map $d: A \to A$ such that $d\mu_i = \mu_i (d\otimes 1 + 1\otimes d)$. Given an algebra A from Var and its derivation d, let us introduce new operations on the space A: $\mu_i^{\succ} = \mu_i(d \otimes 1)$, $\mu_i^{\prec} = \mu_i(1 \otimes d)$. Denote by $A^{(d)}$ the *derived* algebra, i.e., the same underlying space A with operations

$\mu_i^{\succ}, \mu_i^{\prec}$. The class of all derived algebras generates the *derived variety* denoted by DVar.

Let Com be the variety of associative and commutative algebras with one multiplication μ(x,y)=xy. Then, as it was noted in [1], for every A$\in$Com its derived algebra satisfies (1) and (2) relative to the operation μ_i (x,y) = x ○ y. In [3], it was actually shown that DCom=Nov. In general [4], if the variety Var is defined by multilinear identities (the corresponding operad is also defined Var), then

$$\mathrm{DVar} = \mathrm{Var} \circ \mathrm{Nov}$$

where ○ stands for the Manin white product of operads.

It was proved in [5] that not just free but every Novikov algebra can be embedded into a derived commutative algebra. A similar question concerning an arbitrary variety Var has a negative solution in general: there exist such varieties Var (defined by binary quadratic operads) that not every DVar-algebra embeds into an appropriate derived Var-algebra.

We consider another generalization of Novikov algebras, namely, Novikov conformal algebras.

Lie conformal algebras were introduced by V. Kac [6] as a formal tool for describing the properties of the singular part of the operator product expansion (OPE) in 2-dimensional conformal field theory. A categorial approach to the definition of a conformal algebra leads us naturally to the notions of associative, commutative, and Novikov conformal algebras. In particular, the class of Gel'fand–Dorfman bialgebras and Novikov–Poisson algebras are closely related to Lie and Novikov conformal algebras, respectively.

We are studying the embedding problem for Novikov conformal algebras. As a result, we prove the following

Theorem. For every finitely generated Novikov conformal algebra V there exists a commutative conformal algebra C with a conformal derivation D such that V embeds into the derived conformal algebra $C^{(D)}$.

In the case of infinitely generated conformal algebras, we construct a counterexample to the above-mentioned theorem.

References

[1]. Gelfand I. M.; Dorfman, I. Y. Hamilton operators and associated algebraic structures. Functional analysis and its application 13, 13 -30 (1979).

[2] Balinskii, A. A.; Novikov. S. P. Poisson brackets of hydrodynamic type, Frobenius algebras and Lie algebras. Sov. Math. Dokl. 32, 228 - 231 (1985).

[3] Dzhumadil'daev, A. S.; Lofwall, C. Trees, free right-symmetric algebras, free Novikov algebras and identities. Homology, Homotopy Appl., 4(2), 165 - 190 (2002).

[4] Kolesnikov, P. S.; Sartayev, B.; Orazgaliev, A. Gelfand–Dorfman algebras, derived identities, and the Manin product of operads. J. Algebra {539}, 260 - 284 (2019).

[5] Bokut, L. A.; Chen, Y.; Zhang, Z. Gröbner – Shirshov bases method for Gelfand–Dorfman –Novikov algebras. J. Algebra Appl., 16(1), 1750001, 22 pp. (2017).

[6] Kac, V. G. Vertex Algebras for Beginners. Univ. Lect. Ser., 10, Am. Math. Soc., Providence, RI (1998).

Rosa Navarro

(University of Extremadura, Spain)

Solvable Lie and Leibniz superalgebras with a given nilradical.

Abstract: Throughout this work we show that under certain conditions the method for describing solvable Leibniz (resp. Lie) algebras with given nilradical by means of non-nilpotent outer derivations of the nilradical is also applicable to the case Leibniz (resp. Lie) superalgebras. Moreover, after having established the general method for Lie and Leibniz superalgebras, we classify all the solvable superalgebras on a very important class of each of them, that is, those with nilradical of maximal nilindex. Note that for $(n + m)$-dimensional superalgebras this maximal nilindex is $n + m - 1$ in the Lie case and $n + m$ in Leibniz.

Jiefeng Liu

(Northeast Normal University, China)

Differential calculus on Lie conformal algebroids.

Abstract: In this talk, we first recall some constructions of Lie conformal algebroids. Then we introduce the representations, dual representations and cohomology of Lie conformal algebroids. Next the differential calculus on Lie algerbroids is studied. Finally, we give the isomorphism between the cohomology of Lie conformal algebroids and de Rham complex on algebra of differential functions.

19th September, 2024

Vyacheslav Futorny

(Southern University of Science and Technology, China)

Representations of map Lie superalgebras.

Abstract: We will discuss the classification of Harish-Chandra modules over classical map superalgebras. The classification problem reduces to the classification of cuspidal bounded modules over cuspidal map superalgebras via a parabolic induction, and any such simple module is isomorphic to evaluation module. This is a joint work with L. Calixto and H. Rocha.

Amir Fernández Ouaridi

(University of Cadiz, Spain)

Abelian subalgebras and ideals of maximal dimension in algebras.

Abstract: Abelian subalgebras and ideals of Lie and Leibniz algebras play a key role in their structure. Of particular interest are those abelian subalgebras and ideals of maximal dimension. For their study, it is natural to introduce the functions α and β, which correspond to the dimension of an abelian subalgebra and ideal of maximal dimension, respectively. In our talk, we will discuss the principal findings concerning these two invariants for Lie and Leibniz algebras, including various relationships between α and β, some characterization of Lie and Leibniz algebras based on the codimension of their abelian subalgebras,

particularly for codimension one and two, and the explicit computations of α and β for some distinguished examples.

Farrukh Mashurov
(Southern University of Science and Technology, China)

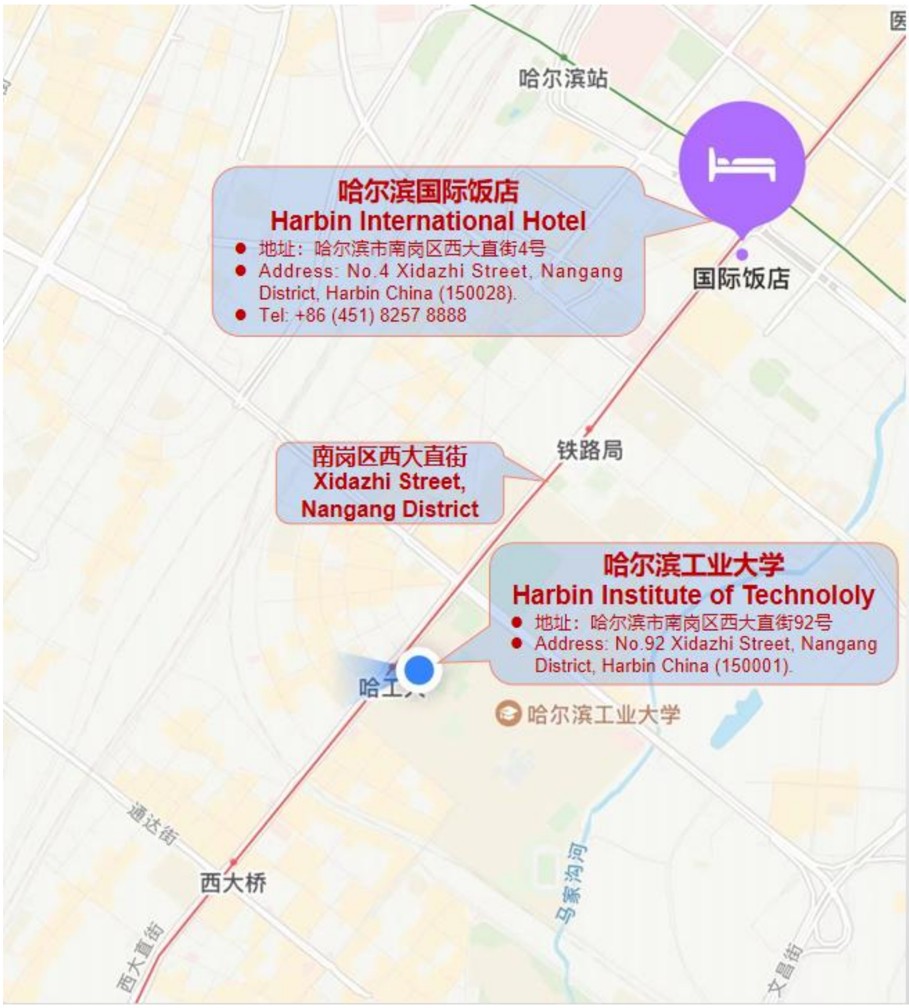
On the contact and transposed Poisson algebras.

Abstract: This talk presents the main results on the characterization and interplay of contact algebras and transposed Poisson algebras within the associative commutative algebras endowed with a Lie bracket. We establish that an algebra (L, ·, [,]) can be both a contact algebra and a transposed Poisson algebra if it satisfies the identities a[b,c]=aD(b)c-abD(c) and [ab,c]=D(a)bc+aD(b)c-abD(c) for all elements a,b,c ∈ L. This result shows that the structures of contact and transposed Poisson algebras are inherently related in the presence of a differential operator D. Various examples are illustrated as consequences of this result. Furthermore, we investigate the structure of Zinbiel differential algebras and show that they form pre-Lie algebras under the multiplication a * b=d(a)b-ad(b). This result is extended to show that every Zinbiel algebra under the product (a • b=a * b – b*a) is a Lie algebra, and consequently, we construct algebras to be both contact algebra and transposed Poisson algebra.

In addition, using polarisation and depolarisation methods, we establish defining identities for algebras that are both contact and transposed Poisson admissible algebras.

Additional Information

1. **Distance from the airport to HIT:** 45 minutes by car
2. **Distance from the hotel to HIT:** 20 minutes walk
3. **Map:**



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