## 几何分析与非线性偏微分方程研讨会

### 程序册



哈尔滨工业大学数学研究院 2019年5月2日-5日

## 几何分析与非线性偏微分方程研讨会 2019年5月2日-5日

会议地点:哈尔滨工业大学明德楼 201 室

#### 邀请报告专家:

陈世炳 (中国科学技术大学)

陈学长 (南京大学)

葛化彬 (中国人民大学)

华波波 (复旦大学)

金天灵 (香港科技大学)

李东升 (西安交通大学)

李海刚 (北京师范大学)

李奇睿 (澳大利亚国立大学)

马 辉 (清华大学)

盛 利 (四川大学)

宋 翀 (厦门大学)

孙 伟 (上海科技大学)

王克磊 (武汉大学)

王 鹏 (福建师范大学)

王作勤 (中国科学技术大学)

夏 超 (厦门大学)

徐国义 (清华大学)

杨 翎 (复旦大学)

周 斌 (北京大学)

朱苗苗 (上海交通大学)

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# 会议日程

### 5月3日 上午

主持人: 关波			
09:00-9:50	金天灵 Solutions of Some Monge-Ampere Equations with Degeneracy or Singularities		
9:50-10:15	合影、茶歇		
10:15-11:05	王克磊 Finite Morse Index Solutions of the Allen-Cahn Equation		
11:10-12:00	杨翎	极小子流形的 Gauss 映照值分布问题	
12:00	午餐: 西苑宾馆		

### 5月3日 下午

主持人: 薛小平			
14:00-14:50	李海刚	Babuska Problem in Composite Materials and its Application in Finsler Geometry	
14:55-15:45	Symmetric Minimal Surfaces in S^3 as 王鹏 Conformally-Constrained Willmore Minimizers in S^n		
15:45-16:05	茶歇		
16:05-16:55	葛化彬     双曲多面体的刚性与实现		
17:00-17:50	马辉 Self-Similar Solutions and Umbilic Hypersurfaces		
18:00	晚餐: 西苑宾馆		

## 5月4日 上午

主持人: 王明新				
09:00-9:50	李东升 Existence and Boundary Asymptotic Behavior of Hessian Equations			
9:50-10:10	茶歇			
10:10-11:00	盛利 A Bernstein Theorem and A Liouville Theorem			
11:05-11:55	宋翀 Hessian Estimates for Convex Solutions to Quadration			
12:00	午餐: 西苑宾馆			

## 5月4日 下午

主持人: 隋哲楠			
14:00-14:50	王作勤 On Weyl Laws		
14:55-15:45	李奇睿 A Class of Monge-Ampere Equations on the Sphere		
15:45-16:05	茶歇		
16:05-16:55	周斌 Moser-Trudinger Type Inequality and Regularity of the Complex Monge-Ampere Equation		
17:00-17:50	陈学长 Rigidity Theorems of the Obata Equation on Compact Manifolds with Boundary		
18:00	晚宴: 西苑宾馆		

### 5月5日 上午

主持人: 张超			
09:00-9:50	朱苗苗	Geometric Analysis of a Mixed Elliptic-Parabolic Conformally Invariant Boundary Value Problem	
9:50-10:10	茶歇		
10:10-11:00	徐国义 The Analysis and Geometry of Isometric Embedding		
11:05-11:55	华波波 图上的 p-Laplace 算子特征值和特征函数		
12:00	午餐: 西苑宾馆		

## 5月5日 下午

主持人: 矫贺明			
14:00-14:50	陈世炳 Free Boundary Problem in Optimal Transportation		
14:55-15:45	孙伟	The Parabolic Flows for Complex Quotient Equations	
15:45-16:05	茶歇		
16:05-16:55	夏超	Symmetrization with respect to Mixed Volumes	
17:30	晚餐: 西苑宾馆		

### 报告题目与摘要

#### Free Boundary Problem in Optimal Transportation

陈世炳 中国科学技术大学

**Abstract.** Caffarelli and McCann proved that when two domains are strictly convex and disjoint, the associated densities are bounded from below and above, then the free boundary arising in the optimal transport is  $C^{1,\alpha}$ . I will discuss some partial results concerning the higher order regularity.

#### Rigidity Theorems of the Obata Equation on Compact Manifolds with Boundary

陈学长 南京大学

**Abstract.** We study the Obata equation with Robin boundary condition, as well as non-vanishing Neumann boundary condition, and can extend Obata's rigidity theorem to compact manifolds with boundary. As an application, we consider the equality case of the first eigenvalue estimate of the Laplacian with Robin boundary condition under some natural geometric hypothesis. This is joint with Mijia Lai and Fang Wang.

#### 双曲多面体的刚性与实现

葛化彬 中国人民大学

**摘要**. 多面体的刚性和实现问题由来已久,欧几里得的《几何原本》中证明了只有五种正多面体,Cauchy 在博士论文中证明,欧氏空间的有限凸多面体完全被组合结构以及表面多边形的几何确定。利用 Andreev 关于双曲多面体的刚性与实现结果,Thurston 证明了 3 维 Haken 流形的双曲化定理。

本次报告,我将介绍用 circle packing、离散共形变换等工具解决刚性问题,以及用离散 Ricci/Yamabe 流寻找 circle packing 以及离散共形变换,进而实现双曲多面体的办法。另外,还会给出无限双曲凸多面体刚性的一些结果,以及实现无限双曲凸多面体的一些猜测。

#### 图上的 p-Laplace 算子特征值和特征函数

#### 华波波 复旦大学

**摘要.** 我们研究图上带 Dirichlet 边界的 p-Laplace 第一特征值和特征函数, 当 p 趋于 1 时,得到关于 1-Laplace 算子的特征值和特征函数的信息。这是与葛化彬、蒋文峰、王丽莉的合作工作。

#### Solutions of some Monge-Ampere Equations with Degeneracy or Singularities

#### 金天灵 香港科技大学

**Abstract.** We will talk about a Liouville type theorems for solutions of some degenerate Monge-Ampere equations. We will also discuss results on existence, regularity, classification, and asymptotic behavior of solutions to some Monge-Ampere equations with isolated and line singularities. This is joint work with J. Xiong.

#### **Existence and Boundary Asymptotic Behavior of Hessian Equations**

#### 李东升 西安交通大学

**Abstract.** In this talk, we will establish the existence of viscosity solutions of Hessian equations with singular right-hand sides and obtain the asymptotic boundary behavior of solutions. The asymptotic results generalize those for Poisson equations and are more precise than obtained from Hopf lemma. We will also establish the existence of large solutions of Hessian equations and obtain a new boundary asymptotic behavior of solutions.

#### Babuska Problem in Composite Materials and its Application in Finsler Geometry

#### 李海刚 北京师范大学

Abstract. In high-contrast composite materials, the stress concentration is a common phenomenon when inclusions are close to touch. It always causes damage initiation. This problem was proposed mathematically by Ivo Babuska, concerning the system of linear elasticity, modeled by a class of second order elliptic systems of divergence form with discontinuous coefficients. I will first review some of our results on upper bound estimates by developing an iteration technique with respect to the energy integral to overcome the difficulty from the lack of maximal principle for elliptic systems. We real the relationship between the blow-up rate, the dimension, and the relative convexity. I will present two very recent results of myself on lower bound estimates and asymptotics of the gradients to show the optimality of the blow-up rates. Finally, I will show that our framework can be applied to investigate the corresponding phenomenon for p-Lapalacian and Fishler Lapalacian in Finsler geometry.

#### A Class of Monge-Ampere Equations on the Sphere

李奇睿 澳大利亚国立大学

**Abstract.** There are a number of geometric problems which can be reduced to the study of Monge-Ampere equations on the sphere, including the Aleksandrov problem, the Minkowski problem, and the more general L\_p dual Minkowski problem introduced recently by Lutwak-Yang-Zhang. In this talk we give a brief discussion on these problems.

#### **Self-Similar Solutions and Umbilic Hypersurfaces**

马辉 清华大学

**Abstract.** In this talk, we report our recent results on hypersurfaces in warped product manifolds, based on joint work with Shanze Gao and Haizhong Li.

Firstly, we prove a uniqueness result for any strictly convex closed hypersurface in  $R^{n+1}$  satisfying  $F - C = \langle X, \nu \rangle$ , called a self-similar solution of the F-curvature flow. Here X is the position vector of the hypersurface, F is a positive homogeneous smooth symmetric function of the principal curvatures, including powers of mean curvature and Gauss curvature, and C is a constant.

We also study self-similar solutions in warped products satisfying  $F-C=g(\lambda(r)\partial_r, \nu)$ . We show that slices are the only closed strictly convex self-similar solutions in the hemisphere for such F. A similar uniqueness result also holds in hyperbolic space  $H^3$  for Gauss curvature F and  $C \ge 1$ .

Finally, by using a new integral formula or Brendle's Heintze-Karcher type inequality, we present some new characterizations of umbilic hypersurfaces in a wide class of warped product manifolds. These results can be viewed as generalizations of the classical Jellet-Liebmann theorem and the Alexandrov theorem in Euclidean space.

#### A Bernstein Theorem and A Liouville Theorem

盛利 四川大学

**Abstract.** In this talk we consider a class of fourth order PDE. We establish a Bernstein Theorem by affine techniques. Next we talk a Liouville Theorem on the PDE det(fij) = 1 under some assumption.

#### **Hessian Estimates for Convex Solutions to Quadratic Hessian Equation**

宋翀 厦门大学

**Abstract.** We derive Hessian estimates for almost convex solutions to the quadratic Hessian equation, i.e. sigma-2 equation, by a compactness argument. This is a joint work with Matt McGonagle and Yu Yuan.

#### The Parabolic Flows for Complex Quotient Equations

孙伟 上海科技大学

**Abstract.** The parabolic flow method can be applied to solving complex quotient equations on closed Kahler manifolds. As a result, we solve the complex quotient equations.

#### Finite Morse Index Solutions of the Allen-Cahn Equation

王克磊 武汉大学

**Abstract.** In this talk I will discuss the structure of finite Morse index solutions of the Allen-Cahn equation. Several tools are used in this study, including the curvature estimate of Schoen type, the uniform second order estimates of clustering interfaces and its connection with Toda systems. The talk is based on joint works with Changfeng Gui and Juncheng Wei.

# Symmetric Minimal Surfaces in S^3 as Conformally-Constrained Willmore Minimizers in S^n

王鹏 福建师范大学

**Abstract.** The Willmore conjecture states that the Clifford torus minimizes uniquely the Willmore energy \int (H^2+1) dM among all tori in S^3, which is solved recently by Marques and Neves in 2012. For higher genus surfaces, it was conjectured by Kusner that the Lawson minimal surface, \xi\_{m,1}: M-->S^3, minimizes uniquely among all genus m surfaces in S^n. The conjecture reduces to the Willmore conjecture for tori if m=1, since \xi\_{1,1} is the Clifford torus. In this talk, we will prove this conjecture under the assumption that the (conformal) surfaces in S^n have the same conformal structure as \xi\_{m,1}.

#### On Weyl Laws

#### 王作勤 中国科学技术大学

**Abstract.** Weyl law relates the asymptotic behavior of eigenvalues of certain operators with the geometric/analytic/dynamic behavior of the background manifold. It was first discovered by H. Weyl in 1911 for the Dirichlet-Laplace eigenvalues of bounded regions and then extended/strengthened by many mathematicians to various general settings. In this talk I will briefly describe these connections and discuss some recent work.

#### Symmetrization with respect to Mixed Volumes

#### 夏超 厦门大学

**Abstract.** It is well known that the Schwarz symmetrization of a function diminishes the Dirichlet integral. This is the so-called Polya-Szego Principle. An similar symmetrization with respect to quermassintegral, which has been introduced by Talenti and Tso, diminishes the Hessian type integral. In this talk I will introduce my recent joint work with Della Pietra and Gavitone, on new symmetrization with respect to mixed volume or anisotropic curvature integral.

#### The Analysis and Geometry of Isometric Embedding

#### 徐国义 清华大学

**Abstract**. In 1950's, Nash-Kuiper built up the C^1 isometric embedding for any surface into \mathbb{R}\^3, this can be viewed as analysis side of isometric embedding. On the other hand, there is obstruction for the existence of C\^2 isometric embedding of surface into \mathbb{R}\^3 known since Hilbert, which reflects the geometry flavor of isometric embedding. What's happening from C\^1 to C\^2 (from analysis to geometry)? We will present our partial progress along this direction. The talk will be accessible to audience with basic knowledge of PDE and Riemannian geometry.

#### 极小子流形的 Gauss 映照值分布问题

#### 杨翎 复旦大学

**摘要.** 极小子流形的 Gauss 映照值分布问题与 Grassmann 流形的几何性质、极小子流形第二基本形式的代数性质以及椭圆型偏微分方程的 Liouville 性质密切相关。在本报告中,我们将简述本人以及合作者在本领域的成果以及思想。

#### Moser-Trudinger Type Inequality and Regularity of the Complex Monge-Ampere Equation

周斌 北京大学

**Abstract.** In this talk, I will first give an introduction on the Sobolev and Moser-Trudinger type inequalities for Monge-Ampere type equations. Then I will present a new proof for the Moser-Trudinger type inequality for the complex Monge-Ampere equation. As an application, we establish a PDE approach to the regularity of the complex Monge-Ampere equation.

#### Geometric Analysis of a Mixed Elliptic-Parabolic Conformally Invariant Boundary Value Problem

朱苗苗 上海交通大学

**Abstract.** In this talk, we discuss a systematic treatment of the geometric analysis of a variational problem originating from the nonlinear supersymmetric sigma model of QFT. We develop a general spectrum of methods to solve the corresponding boundary value problem and to analyze the qualitative behavior of the solutions. Our works build upon the impressive advances and vast developments over several decades in the field of two dimensional geometric conformally invariant problems since the fundamental works of Jonathan Sacks-Karen Uhlenbeck, Leon Simon and Michael Struwe etc. in the 1980s.

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