



哈爾濱工業大學  
HARBIN INSTITUTE OF TECHNOLOGY

# 2023 HARBIN MATH LOGIC CONFERENCE



## *Program*

Institute for Advanced Study in Mathematics  
August 17–19, 2023



The 2023 Harbin Math logic Conference is hosted by the Institute for Advanced Study in Mathematics of Harbin Institute of Technology, on August 18-19,2023. The conference aims to bring together Chinese logicians to discuss the frontiers of math logic and promote potential collaboration; a number of invited talks will present recent progress on various topics in set theory, general topology, model theory, and recursion theory.

## Venue for Harbin participants

Room 201, Ming De Building, Harbin Institute of Technology

## Organizing committee

Haosui Duanmu Harbin Institute of Technology

Xiao Xiong Harbin Institute of Technology

Quanhua Xu Harbin Institute of Technology and Université de Franche-Comté

## Links

Conference website: <http://im.hit.edu.cn/2023/0801/c8389a321695/page.htm>

Institute for Advanced Study in Mathematics of HIT: <http://im.hit.edu.cn/en/>

# Timetable

## August 18

9:00–9:10	<b>Welcome Remark by Liang Yu</b>	
9:10–9:30	<b>Group Photo</b>	
9:30–10:20	<b>Yue Yang</b> National University of Singapore	Some Generalizations of Posner-Robinson Theorem
10:20–11:10	<b>Yinhe Peng</b> Chinese Academy of Sciences	An inner approach with continuum large
11:10–11:30	<b>Break</b>	
11:30–12:20	<b>Zhaokuan Hao</b> Fudan University	Concept and Set
12:20–14:00	<b>Break</b>	
14:00–14:50	<b>Liang Yu</b> Nanjing University	AD vs AC
14:50–15:40	<b>Yuping Shen</b> Sun Yat-sen University	Computationally Hard Problems for Some Boolean Models of Computation
15:40–16:00	<b>Break</b>	
16:00–16:50	<b>Xinyu Liu</b> Beijing Jiaotong University	A Nonstandard Proof of De Finetti's Theorem for Multinomial Distribution
16:50–17:40	<b>Shichang Song</b> Beijing Jiaotong University	Projective Fraisse limits and profinite groups

## August 19

9:00–9:50	<b>Liuzhen Wu</b> Chinese Academy of Sciences	Generalized continuum hypothesis at a finite interval above strongly compact cardinal
9:50–10:40	<b>Guozhen Shen</b> Wuhan University	Cantor's theorem may fail for finitary partitions
10:40–11:00	<b>Break</b>	
11:00–11:50	<b>Jialiang He</b> Sichuan University	On the extendability to $\pi_3^0$ ideals and Katetov order

# List of Abstracts – Talks

## Concept and Set

*Zhaokuan Hao (Fudan University)*

11:20-12:10 August 18

The systematic search for a proper foundation for mathematics at the end of nineteenth century resulted in a variety of programs, the most important of which were Cantor's set theory and Frege's logicism. The fundamental difference between Cantor's program and Frege's is that the former takes set as the basic concept while the latter takes concept. In this talk I try to argue that logicism, which was once considered a failure, is closer to the truth of the universe of set from the perspective of the development of set theory today.

## On the extendability to $\pi_3^0$ ideals and Katetov order

*Jialiang He (Sichuan University)*

11:00-11:50 August 19

we show that there is a borel ideal such that it's neither extendable to any  $\pi_3^0$  ideal nor above the ideal  $\text{Fin} \times \text{Fin}$  in the sense of Katetov order, answering a question from M. Hrusak.

## A Nonstandard Proof of De Finetti's Theorem for Multinomial Distribution

*Xinyu Liu (Beijing Jiaotong University)*

16:00-16:50 August 18

In this talk, we give a nonstandard proof of de Finetti's theorem for multinomial distribution which states that every exchangeable sequence with three values is a mixture of independent sequences with respect to a measure. This proof is based on a paper by Irfan Alam with the title *A Nonstandard Proof of De Finetti's Theorem for Bernoulli Random Variables*. In his paper, Alam defined a random variable  $Y_N$  as the frequency of 1 in an exchangeable sequence of Bernoulli random variables, where  $N$  can take the value of a positive infinite nonstandard real number. Then, the rest of the paper is devoted to prove that the measure required in de Finetti's theorem for Bernoulli random variables can be constructed via the pushforward measure of the Loeb measure associated with the internal probability measure induced by  $Y_N$ . By the analogy of the above method, we can extend the theorem from dimension two to dimension three.

## An inner approach with continuum large

*Yinhe Peng (Chinese Academy of Sciences)*

10:20-11:10 August 18

We introduce a new inner approach construction of  $\gamma$ -sets of reals. A set is a  $\gamma$ -set if every  $\omega$ -cover has a  $\gamma$ -subcover. An open cover is an  $\omega$ -cover if for every finite subset, there is one open set containing it. An infinite open cover is a  $\gamma$ -cover if every point is contained in all but finitely many open sets. Applying the method, we construct consistently with continuum large,  $\gamma$ -sets  $X, Y \subset \mathbb{R}$  such that  $X \times \{0\} \cup Y \times \{1\}$  is not a  $\gamma$ -set.

## Cantor's theorem may fail for finitary partitions

**Guozhen Shen (Wuhan University)**

9:50-10:40 August 19

A partition is finitary if all its members are finite. For a set  $A$ ,  $\mathcal{B}(A)$  denotes the set of all finitary partitions of  $A$ . It is shown consistent with ZF (without the axiom of choice) that there exist an infinite set  $A$  and a surjection from  $A$  onto  $\mathcal{B}(A)$ . On the other hand, we prove some ZF theorems concerning  $\mathcal{B}(A)$ , among which are the following: (1) For all infinite sets  $A$ , if there is a finitary partition of  $A$  containing no singletons, then there are no surjections from  $A$  onto  $\mathcal{B}(A)$  and no finite-to-one functions from  $\mathcal{B}(A)$  to  $A$ ; (2) For all infinite sets  $A$  and all natural numbers  $n$ ,  $|A^n| < |\mathcal{B}(A)|$ ; (3) For all infinite sets  $A$ ,  $|\text{seq}^{1-1}(A)| \neq |\mathcal{B}(A)| \neq |\text{seq}(A)|$ , where  $\text{seq}(A)$  and  $\text{seq}^{1-1}(A)$  denote the sets of all finite sequences and all finite sequences without repetition of elements of  $A$ , respectively.

## Computationally Hard Problems for Some Boolean Models of Computation

**Yuping Shen (Sun Yat-sen University)**

14:50-15:40 August 18

Showing a problem is *hard* for a model of computation is one of the most challenging tasks in theoretical computer science, logic and mathematics. For example, it remains beyond reach to find an *explicit* problem that cannot be computed by polynomial size propositional formulas (PF). As a boolean model of computation, logic programs (LP) under answer set semantics are as expressive as PF, and also NP-complete for satisfiability checking. In this paper, we show that the Parity problem is hard for LP, i.e., such a computation requires *exponential* size programs. The main result is obtained by applying probabilistic method from circuit complexity. Based on the main result, we further generalize some classes of hard problems for LP, and give a separation map for a family of boolean computation models.

## Projective Fraisse limits and profinite groups

**Shichang Song (Beijing Jiaotong University)**

16:50-17:40 August 18

We show that the projective Fraisse limit of the class of finite groups is the free profinite group on a countably infinite set converging to 1. Joint work with Sulin Hu.

## Generalized continuum hypothesis at a finite interval above strongly compact

## cardinal

**Liuzhen Wu (Chinese Academy of Sciences)**

9:00–9:50 August 19

We report some recent progress on the project to determine the behavior of continuum function on strongly compact cardinal. In the previous talk, we show that one can get the consistency of continuum hypothesis at a strongly compact cardinal from one strongly compact cardinal. In this talk, we show that we can push the previous argument to get Generalized continuum hypothesis at a finite interval above strongly compact cardinal from one strongly compact cardinal.

## Some Generalizations of Posner-Robinson Theorem

**Yue Yang (National University of Singapore)**

9:30–10:20 August 18

The original Posner-Robinson Theorem can be stated as, using their own words in [PR 1981], “...any nonrecursive degree below  $0'$  can be joined to  $0'$  by a degree strictly below  $0'$ .” It turns out that this seemingly technical theorem about local degrees revealed some deepest connections between the jump and join operators in Turing degrees, and had many important applications in global degree theory, for instance, Shore and Slaman’s result on the definability of Turing jump [SS 1999].

In [PR 1981], Posner and Robinson in fact proved something more general. They showed that a uniform recursive family of nonzero degrees below  $0'$  can be joined by a single degree to  $0'$ , and they also combined the above result with cone avoiding. In this talk, I will talk about an alternative proof of the result above (without the restriction of below  $0'$ ), and establish a necessary and sufficient condition on degrees  $a_i$  ( $i = 1, 2, \dots$ ) so that a single degree  $g$  can join  $a_i$  to the  $i$ -th jump of  $g$ .

This is a joint work (and some work in progress) with Ted Slaman (UC Berkeley), Frank Stephan (NUS), Samuel Tanuwijaya (NUS) and Liang Yu (Nanjing U).

References:

[PR 1981] David B. Posner, and Robert W. Robinson, *Degrees joining to  $0'$* , JSL 1981, pp.714-722.

[SS 1999] Richard A. Shore and Theodore A. Slaman, *Defining the Turing Jump*, Mathematical Research Letters 1999, 711-722.

## AD vs AC

**Liang Yu (Nanjing University)**

14:00–14:50 August 18

We discuss some relationships between axiom of determinacy and choice.